EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng. Struct. Dyn. **28**, 1255–1271 (1999)

INFLUENCE OF GROUND MOTION INTENSITY ON THE EFFECTIVENESS OF TUNED MASS DAMPERS

RICARDO SOTO-BRITO AND SONIA E. RUIZ*
Instituto de Ingenieria, UNAM, Apdo, Postal 70-472, Coyoacán 04510 México, D.F., Mexico

SUMMARY

The effectiveness of Tuned Mass Dampers (TMD) on buildings subjected to moderate and high-intensity motions is analysed. First, the response of a 22-storey four-bay reinforced concrete non-linear frame with a TMD is studied for motions with different intensities. Several values of the relevant parameters are assumed in the analyses. Then, equivalent single-degree-of-freedom systems with TMDs and without them are defined and analysed under the action of ground motions with intensities associated with different return intervals at the site where the structures are located. Vulnerability curves for the systems are obtained based on the probabilities of reaching two different performance limit states. The expected annual rate of exceedance of each limit state is calculated. The results show that the effectiveness of TMDs is higher for systems with small non-linearity produced by small and moderate earthquakes, than for systems with high non-linear behaviour, generally associated with high-intensity motions. Some recommendations about the applicability of TMD are given. Copyright © 1999 John Wiley & Sons Ltd.

KEY WORDS: tuned mass dampers; expected failure rates; vulnerability curves

1. INTRODUCTION

Tuned Mass Dampers (TMD) are used efficiently to control the vibrations of mechanical systems, bridges, towers, etc., as well as to reduce the dynamic structural response caused by the wind action; however, there is no consensus among engineers and researchers about their effectiveness in controlling the structural responses due to earthquakes.

Villaverde and Koyoama,[1] and Bernal[2] have shown that the base shear of a 2 s period *linear* building subjected to the motion recorded on soft soil in Mexico City during the September 1985 earthquake (SCT-85) is reduced to about 40 per cent when a properly designed TMD is placed on its top. This means that the peak ordinate of the acceleration spectrum for 0·05 damping can be reduced from $1g$ ($g =$ gravity) to $0{\cdot}6g$; unfortunately this value would still give place to a very high design lateral forces, and consequently to high construction costs; it would be inappropriate to use it for design purposes.

The foregoing information stimulated Ruiz and Esteva[3] to briefly explore the possible advantages of using TMDs on structures designed to develop non-linear behaviour. The conclusions

*Correspondence to: Sonia E. Ruiz, Instituto de Ingeniería, UNAM, Apdo, Postal 70-472, Coyoacán 04510 México, D.F., Mexico.

CCC 0098–8847/99/111255–17$17·50

Received 28 October 1998
Accepted 21 May 1999

were that the reductions in the structural response that can be obtained by adding a TMD to a non-linear system become small as the non-linearity increases.

The non-linear behaviour of building constructions subjected to earthquake motions highly depends on the design specifications of the structural elements, as well as on the intensity of the motion at the site where the building is located.

In this study it is shown that TMDs can be useful for reducing the lateral drift of buildings subjected to moderate earthquakes, but not for high-intensity motions.

This is illustrated by analysing several 22-storey four-bay reinforced concrete frames, as well as equivalent Single-Degree-Of-Freedom systems (SDOF) subjected to motions with different intensities. Each structure is analyzed with a TMD on its top and without it. The intensities of the motions cover a wide range of values, associated with return intervals varying from 5 to 100 years. From the vulnerability curves associated with the probabilities of reaching different limit states, it is concluded that the effectiveness of a TMD decreases as the intensity of the motion and the resulting nonlinearity of the system increase.

2. EARTHQUAKE MOTIONS

Two different acceleration records are used in this study as excitations, representative of moderate and high-intensity motions, respectively. Both were recorded at the parking lot of the Ministry of Communications and Transportation (SCT) in Mexico City, where the dominant period is about 2 s. The first one corresponds to a seismic event of magnitude $M_s = 6{\cdot}9$, occurred in April 25, 1989 (SCT-89). The second ocurred in September 19, 1985 (SCT-85), with magnitude $M_s = 8{\cdot}1$. Their acceleration spectra for 5 per cent of critical damping are shown in Figure 1. From these accelerograms two families of 50 simulated motions were generated. The evolutionary nature of the intensity and frequency content with time were considered in the mathematical model.[4]

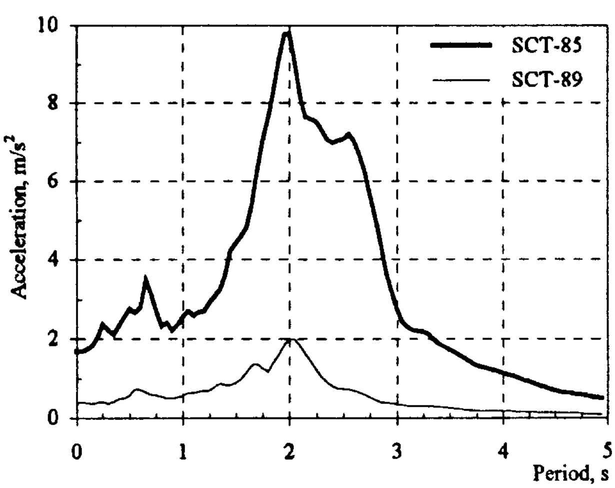

Figure 1. Acceleration spectra of the records used in the study

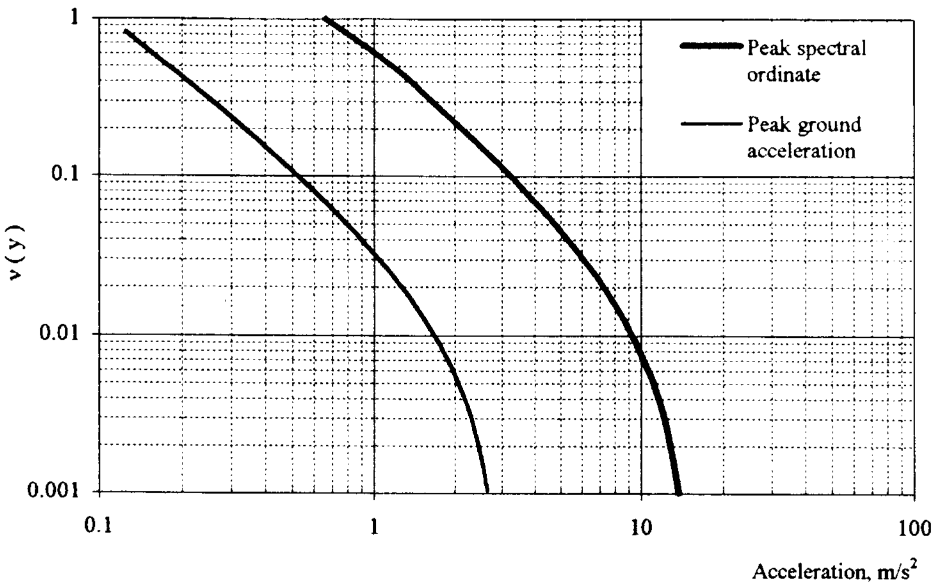

Figure 2. Seismic hazard curves for SCT site

2.1. Expected exceedance rates of intensities

The seismic hazard at the site of interest may be represented by the mean rate of occurrence per unit of time ($\nu(y)$) of motions with intensity equal or higher than y. In this study the following hazard function form is adopted:[5]

$$(y) = Ky^{-r}\left[1 - \left(\frac{y}{y_M}\right)^{\varepsilon}\right], \quad y \leqslant y_M$$

$$\nu(y) = 0, \quad y > y_M$$

where y_M is the maximum intensity expected at the site, r and ε are parameters associated with the intensity distribution, and K is a scale factor.

In this study the intensities of interest are the maximum ground accelerations and the peak spectral ordinates for 5 per cent damping at SCT site.[5] These are shown in Figure 2.

3. ANALYSIS OF 22-STOREY NON-LINEAR FRAMES

The frames analysed in the first part of this study are representative of an office building with a fundamental period of 2 s, having a plant of 28×28 m^2. The height of the first storey is 4·0 m; all other stories are 3·1 m high (see Figure 3). The cross-sections of the girders vary from $0{\cdot}55 \times 0{\cdot}95$ m at the lower levels, to $0{\cdot}45 \times 0{\cdot}85$ m at the top stories. The dimensions of the columns vary from $1{\cdot}20 \times 1{\cdot}20$ m at the first stories, to $0{\cdot}70 \times 0{\cdot}70$ m at the top levels.

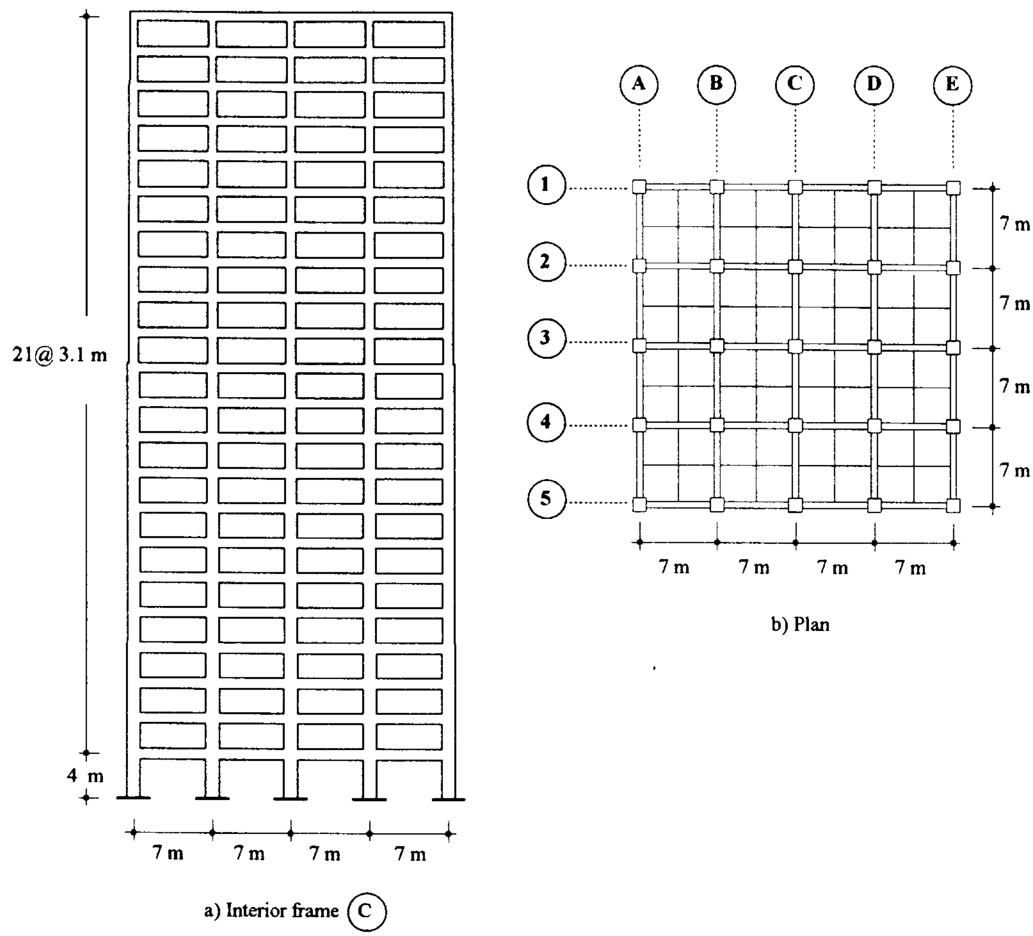

Figure 3. Building geometry

The design was made in accordance with the Mexico City Seismic Design Regulations (1993). A reduction factor $Q = 2$ was applied to the linear acceleration design spectrum, in order to consider the non-linear behaviour. The design base shear was equal to 716 ton. The damping ratio was assumed equal to 5 per cent of critical.

The response of the system without a TMD was compared with that of similar systems with added TMDs at their tops. In each case, the device was represented by an additional equivalent storey. The properties of the TMDs were defined in terms of their mass ratios with respect to that of the main frame ($R_m = 1$, 3 and 5 per cent) their damping values ($\xi = 5$, 20 and 30 per cent of the critical value), and by the ratios of their natural frequencies to that of the frame ($R_\omega = 0{\cdot}6$–$1{\cdot}4$). Figures 4(a)–4(f) show the influence of these variables (R_m, ξ and R_ω) on the maximum displacements of the frames with a TMD, normalized with respect to those without it (D_{WT}/D).

The figures appearing at the left side correspond to the SCT-85 motion (high-intensity earthquake) and those at the right, to the SCT-89 accelerogram (moderate intensity event). The

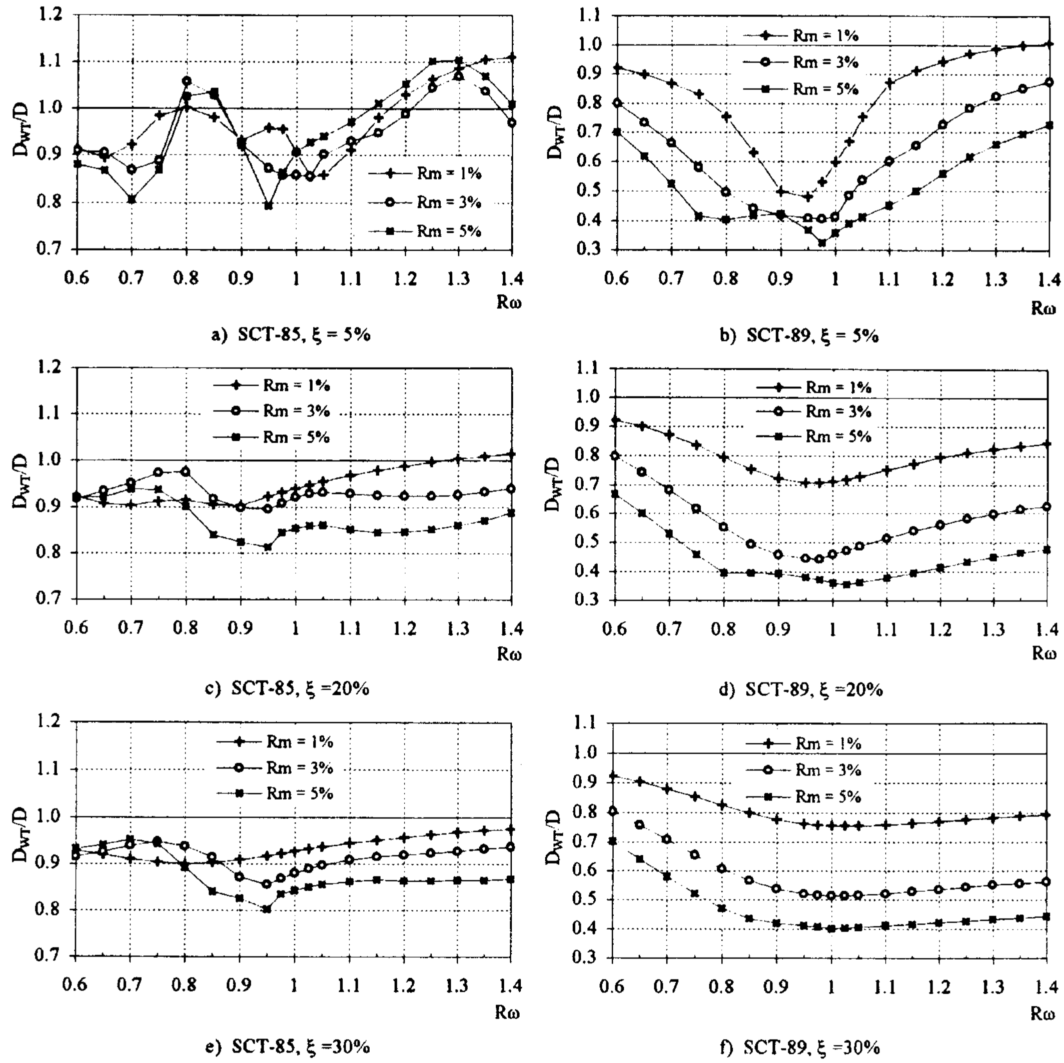

Figure 4. Ratios of maximum displacements of the building with a TMD to those without it (D_{WT}/D). Excitation: SCT-85 (left), and SCT-89 (right). Damping of the TMD: (a–b) $\xi = 5\%$; (c–d) $\xi = 20\%$; (e–f) $\xi = 30\%$

influence of the earthquake intensity on the maximum displacement can be appreciated by comparing the figures at the left and right sides. It is clear that the efficiency of the TMD is higher for the structures subjected to the moderate earthquake (SCT-89).

These figures also show that the influence of the damping of the TMD (ξ) is more significant for systems with smaller values of R_m (1 per cent) than for those with higher values (5 per cent).

It can also be seen that the influence of damping is different for moderate and high-intensity motions.

The increasing of damping (ξ) of the TMD has a clear influence on the maximum roof displacements of frames subjected to moderate intensity earthquakes. The tendencies found for

the cases analysed are as follows: as the damping ξ increases the efficiency of the TMD decreases for frequency ratios R_{ω} close to unity. This is more remarkable for smaller mass ratios R_{m}. On the opposite side, the efficiency of the TMD increases for frequency ratios larger than approximately 1·1 and higher mass ratios R_{m}.

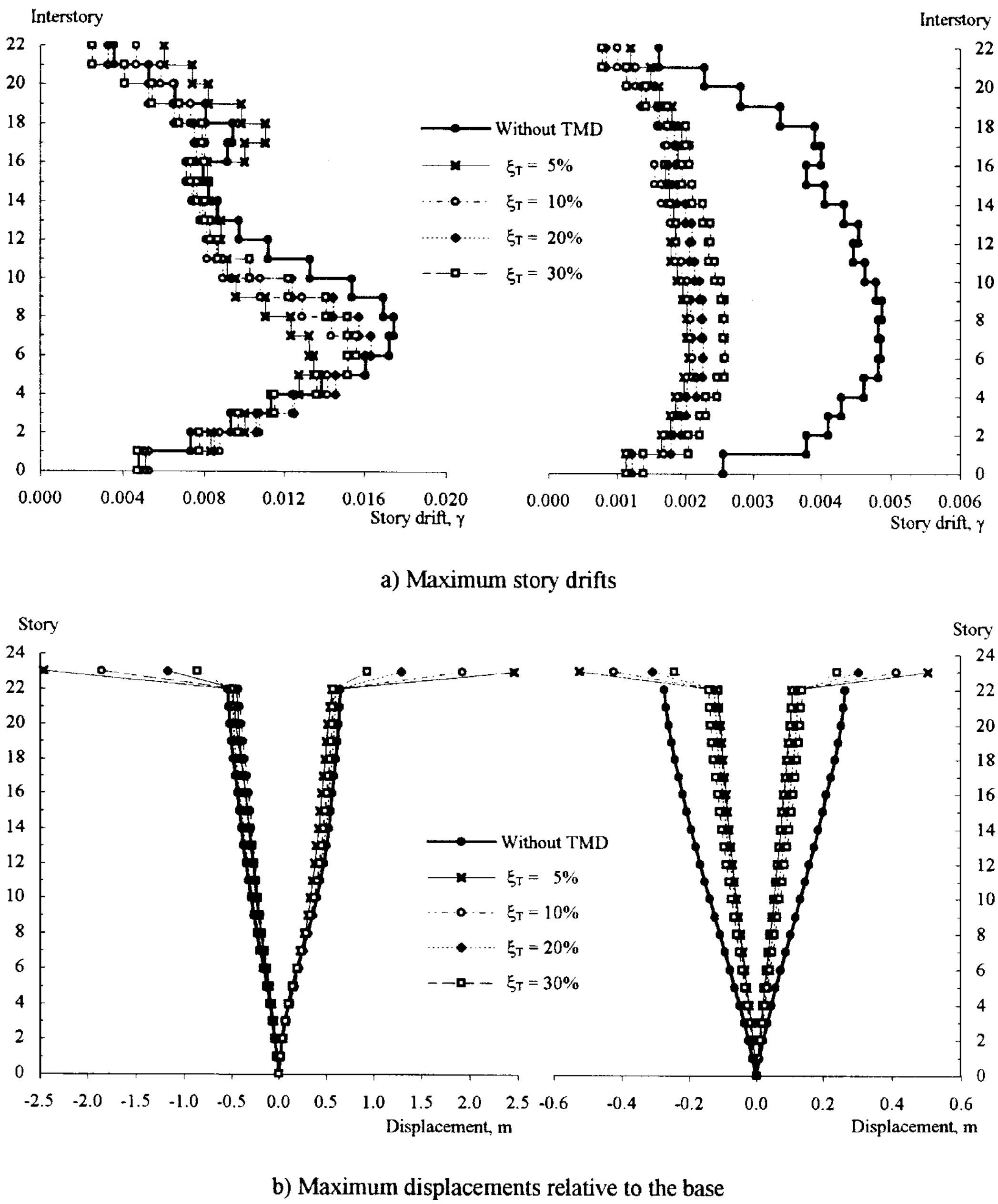

Figure 5. Maximum responses of the systems studied. $R_{\mathrm{m}} = 0{\cdot}3$, $R_{\omega} = 1{\cdot}0$. Excitation: SCT-85 (left) and SCT-89 (right)

For the systems subjected to high-intensity earthquakes, the influence of ξ is qualitatively similar to that described for the previous cases, but less well defined.

For a small damping value ($\xi = 5$ per cent) and high-intensity motion (Figure 4(a)) the frame with a TMD may have maximum displacements larger than those experienced by the systems without it, for frequency ratios R_ω around 0·8–0·85, as well as for those larger than 1·05; however, if the TMD critical damping is increased to 30 per cent (Figure 4(e)) those systems do not present ratios D_{WT}/D larger than 1.

From this, it is clear that the structural response does not necessarily decrease as the device damping increases. This is also illustrated by the following example: let us suppose that the 22-storey frame under study has a TMD on its roof with mass ratio $R_{\mathrm{m}} = 0{\cdot}03$ and frequency rati o $R_\omega = 1{\cdot}0$. Its maximum interstorey drifts and maximum displacements relative to the base associated with device damping values ξ of 5, 10, 20 and 30 per cent of critical are shown in Figures 5(a) and 5(b). Those at the left side correspond to high-intensity motion (SCT-85), and those at the right to moderate motion (SCT-89). From these figures it can be seen that the structural responses of systems with TMD are highly reduced for SCT-89 motion. With the exception of the drifts at the uppermost stories, the largest reductions correspond to the smallest values of ξ. It is observed that at some stories the higher reductions do not correspond to the highest damping assumed ($\xi = 30$ per cent), but to the smallest one ($\xi = 5$ per cent). On the other hand, the maximum deformation of the TMD is smaller as the damping increases (see Figure 5(b)), which constitutes an advantage from the point of view of facilitating construction.

Figures 5(a) and 5(b) also show that the efficiency of the TMD is higher for moderate intensity motions than for high-intensity events.

4. EQUIVALENT SDOF SYSTEMS

The following part of this study deals with the probabilistic response of systems with TMD and without it, subjected to simulated motions, scaled to different return periods. From the results of this section, the vulnerability curves of the systems are constructed.

In order to save computer time it was decided to deal with the equivalent Single-Degree-Of-Freedom (SDOF) systems instead of the 22-storey frames.

Several formulations are available in the literature for the selection of the characteristics of SDOF systems that best represent those of non-linear Multi-Degree-Of-Freedom (MDOF) structures.[6–8] In this study the formulation proposed by Collins *et al.*[8] was adopted for the systems without TMD. A variation of this formulation was adopted for systems with TMD.

4.1. Equivalent systems without TMD

The equation of motion that governs the response of the equivalent SDOF systems without TMD can be expressed as follows:[8]

$$m^*\ddot{D} + c^*\dot{D} + k^*D = -\,l^*a(t)$$

where m^* is the mass, c^* the damping and k^* the stiffness of the SDOF system. These properties are related to those of the detailed system as follows: $m^* = \Psi_2^{\mathrm{T}}\ M\Psi_1$, $c^* = \Psi_2^{\mathrm{T}}\ C\Psi_1$ and

$k^* = KG(D)\Psi_2^{\mathrm{T}} f$; M is the mass matrix, C is the damping matrix of the frame, $l^* = \Psi_2^{\mathrm{T}} MJ$, represents a scale factor of the acceleration motion $a(t)$, D the displacement of the SDOF system, and $J^{\mathrm{T}} = [\,1\;1\;1 \ldots 1]$. f is a prescribed vector of lateral forces which is normalized so that the base shear is unity ($J^{\mathrm{T}} f = 1$). $G(D)$ is a mathematical function which is obtained from a push-over analysis. The variation of the based shear V with the roof displacement D is expressed as $V = KG(D)$, where K is the slope of the initial portion of the curve.

The vector Ψ_1 is a normalized displacement profile obtained from a push-over analysis. The amplitude of this vector at the top of the main system is taken equal to unity. Two alternative assumptions concerning Ψ_2 were considered in this study: $\Psi_2 = \Psi_1$ and $\Psi_2 = J$. The first of these assumptions ($\Psi_2 = \Psi_1$) gave place to smaller relative errors associated with maximum roof displacements and maximum storey drifts. Therefore, it was adopted for the rest of this study.

Figures 6(a) and 6(b) show results of the maximum roof displacements and maximum storey drifts associated with the equivalent SDOF system (vertical axis), and with the 22-storey frame model (horizontal axis); both figures correspond to systems without TMD. They were obtained from 19 simulated accelerograms of the SCT-85 motion, which gave place to non-linear behaviour of the structures. For this case the vector $\Psi_1 = \Psi_2$ was that associated with a global drift of 0·01.

The maximum interstorey drift ratio ($\delta_{\max}$) of the actual structure is estimated as follows:

$$\delta_{\max} = \left[\frac{\Psi_{1,i} - \Psi_{1,i-1}}{h_i}\right]_{\max} D_{\max}$$

where $D_{\max}$ is the maximum roof displacement and h_i the height of the ith storey.

4.2. Equivalent systems with TMD

The equivalent systems with TMD were obtained under the following assumptions:

1. The dynamic properties (mass, stiffness and damping) of the TMD on the roof of the frame and that on the top of the SDOF system are the same (see Figure 7). The mass of the TMD is equal to 3 per cent of the mass of the frame, the critical damping of the TMD is 20 per cent, and the period of vibration of the TMD coincides with the initial period of the main system ($T = 2$ s).
2. The form of $\Psi_1 = \Psi_2$ was obtained as the vector of mean values of the maximum displacements resulting from a step by step analysis of the 22-storey frame with TMD subjected to 19 simulated accelerograms. Two sets of simulated motions were used. One associated with linear behaviour of the structures (moderate motions), and the other associated with non-linear behaviour of the systems (high-intensity motions).

Figures 6(c) and 6(d) show responses of nonlinear systems with TMD. They correspond to maximum roof displacements (left) and maximum interstorey drifts (right), predicted by SDOF models (vertical axis) and by 22-storey frames (horizontal axis). Figures 6(e) and 6(f) show the responses of linear systems with TMD. From these figures it is obvious that the linear response of the systems is better predicted than the nonlinear response, as expected.

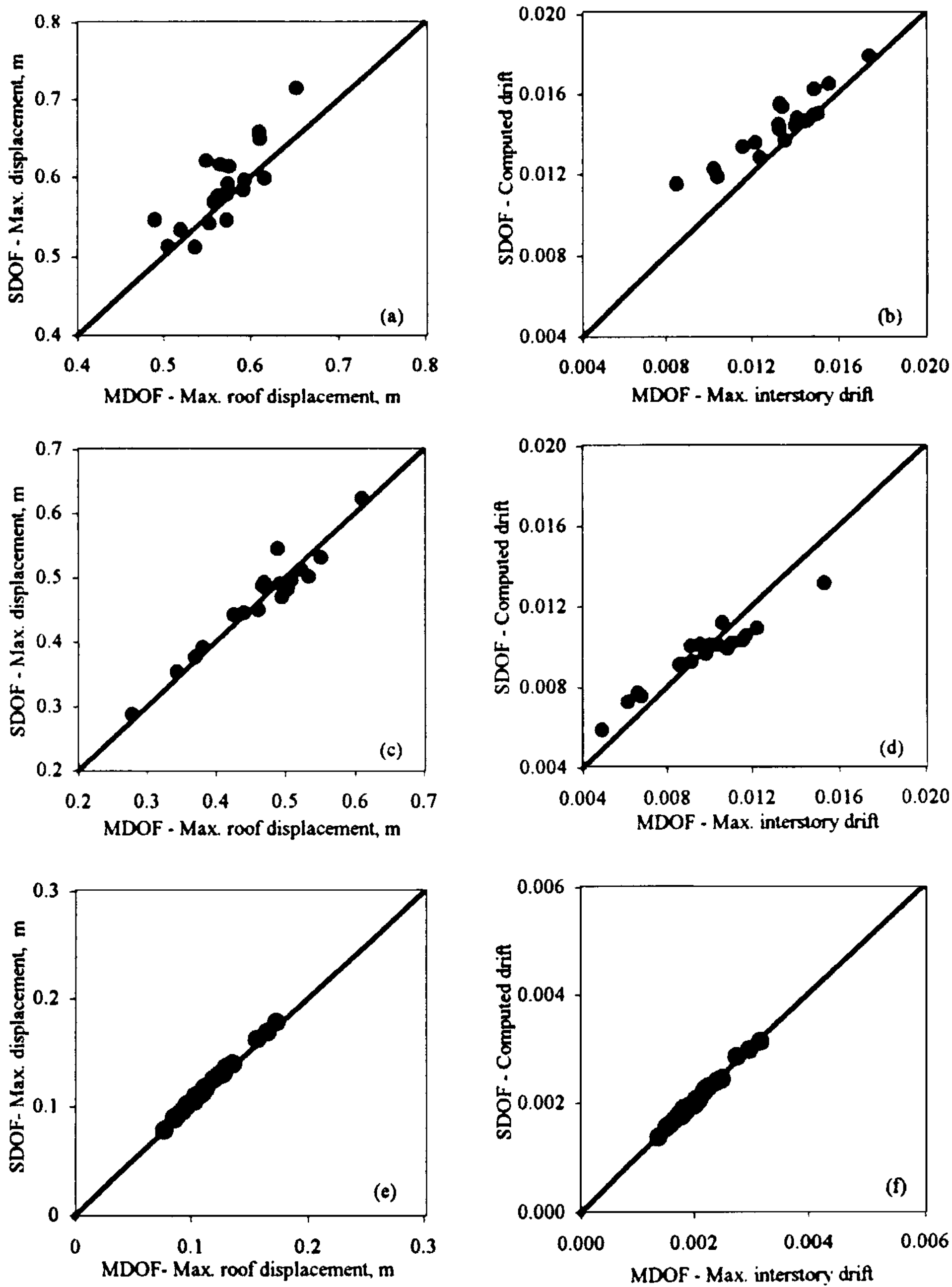

Figure 6. Scatterplot comparing maximum responses (roof displacement and interstorey drift) predicted by SDOF models with responses obtained from a MDOF model of a 22-storey frame

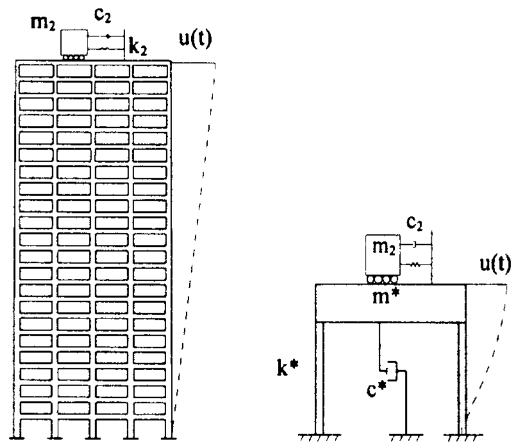

Figure 7. Parameters of the real and the equivalent systems

5. EQUIVALENT SDOF SYSTEMS SUBJECTED TO MOTIONS OF DIFFERENT INTENSITIES

The equivalent systems obtained as described in the previous section were analysed under the action of simulated motions with different intensities. They correspond to return intervals T_R of 25 to 1000 years, and $T_R = 5{\cdot}5-20$ years. The time histories used were scaled versions of the SCT-85 record, for the first group of return intervals, and of the SCT-89 record, for the second group. The scale factors applied to the real and to the simulated accelerograms were based on the seismic hazard function for the maximum ordinates of the acceleration spectra of the SCT site. This function is shown in Figure 2. It represents annual rates of exceedance of different values of the intensity.

For the intensity associated with each return interval, the maximum storey drifts of the SDOF systems with TMD and those without it were calculated using 50 simulated ground motion time histories.

Systems with a seismic coefficient reduction factor $Q = 2$ were analysed first. Then, the resistance of these systems was divided by two. This second case corresponds to $Q = 4$. Finally, systems with linear behavior ($Q = 1$) were analysed.

5.1. Statistics of the response

The statistical parameters (mean m, standard deviation σ and coefficient of skewness g) of the maximum storey drift γ of the systems without TMD and those with it are shown in Tables I, II and III, for values of Q equal to 4, 2 and 1, respectively. Each column of these tables corresponds to a given return interval T_R, in years. The first three rows of the tables refer to systems without TMD, and the next three correspond to systems with TMD. The row at the bottom indicates the ratios of the mean values of the maximum storey drifts of the system with TMD to those without TMD ($m_{\gamma \mathrm{WT}}/m_\gamma$). These ratios give an idea about the response reduction attained with the TMD. In these analyses it was assumed that the structures were able to develop storey drifts larger than those accepted by the seismic code under consideration.

Table I. Statistical parameters (mean, standard deviation and coefficient of skewness) of the maximum storey drift (γ) of buildings without a TMD and with it. Reduction factor $Q = 4$

	T_R (years)	1000	200	100	50	35	25	10
Without TMD	m_γ	0·0199	0·0155	0·0138	0·0117	0·0104	0·00956	0·00715
	σ_γ	0·0051	0·0040	0·0033	0·0024	0·0015	0·00116	0·00064
	g_γ	1·0100	1·2114	1·0564	1·1102	0·7597	0·45707	1·11486
With TMD	$m_{\gamma\mathrm{WT}}$	0·0192	0·0142	0·0122	0·0103	0·0091	0·00841	0·00563
	$\sigma_{\gamma\mathrm{WT}}$	0·0053	0·0037	0·0032	0·0024	0·0019	0·00160	0·00630
	$g_{\gamma\mathrm{WT}}$	0·9663	0·9608	1·1137	1·1718	0·9979	0·92164	0·75710
Ratio	$m_{\gamma\mathrm{WT}}/m_\gamma$	0·97	0·91	0·89	0·88	0·88	0·88	0·79

Table II. Statistical parameters (mean, standard deviation and coefficient of skewness) of the maximum storey drift (γ) of buildings without a TMD and with it. Reduction factor $Q = 2$

	T_R (years)	100	50	35	25	20	5·5
Without TMD	m_γ	0·0174	0·0159	0·0146	0·0117	0·0091	0·0046
	σ_γ	0·0021	0·0016	0·0009	0·0008	0·0007	0·0005
	g_γ	1·6004	0·4583	0·5969	0·7533	0·0433	0·1471
With TMD	$m_{\gamma\mathrm{WT}}$	0·0147	0·0125	0·0111	0·0082	0·0066	0·0028
	$\sigma_{\gamma\mathrm{WT}}$	0·0023	0·0014	0·0009	0·0015	0·0008	0·0003
	$g_{\gamma\mathrm{WT}}$	0·6021	0·8088	0·2946	0·1215	0·2986	0·4803
Ratio	$m_{\gamma\mathrm{WT}}/m_\gamma$	0·84	0·79	0·77	0·70	0·72	0·61

The tendencies of the structural responses shown in Tables I–III are as follows:

1. The mean values of the maximum drifts of both groups of systems with a TMD and without it decrease as the return interval T_R becomes shorter, as expected.
2. The ratios of the mean values of the maximum storey drifts ($m_{\gamma\mathrm{WT}}/m_\gamma$) for the systems designed with $Q = 4$ and 2 (Tables I and II), are smaller as the return interval T_R decreases. This indicates that the efficiency of the TMD increases as the intensity of the motion is lower, that is, when the behaviour of the systems departs less from linearity.
3. The efficiency of the TMD for the systems designed with $Q = 4$ (Table I) is poor. The reduction of the response varies from 3 to 21 per cent when the return interval varies from 1000 to 10 years.
4. For the linear case ($Q = 1$, Table III) the ratios $m_{\gamma\mathrm{WT}}/m_\gamma$ remain constant (0·58) with T_R, except for $T_R = 5{\cdot}5$ years. In this case the base motions were simulated with parameters of the SCT-89 accelerogram, whereas for the longer return intervals the simulation was based on the SCT-85 record.
5. As the non-linearity of the systems decreases (smaller values of Q) the efficiency of the TMD grows. For example, the ratio $m_{\gamma\mathrm{WT}}/m_\gamma$ for $T_R = 100$ years is equal to 0·89, 0·84 and 0·58 for

Table III. Statistical parameters (mean, standard deviation and coefficient of skewness) of the maximum storey drift (γ) of buildings without a TMD and with it. Reduction factor $Q = 1$

	T_R (years)	200	100	50	35	25	5·5
Without TMD	m_γ	—	0·0217	0·0172	0·0145	0·0130	0·0046
	σ_γ	—	0·0020	0·0013	0·0011	0·0008	0·0005
	g_γ	—	− 0·6343	− 2·895	− 2·948	0·0075	0·1471
With TMD	$m_{\gamma WT}$	1·0153	0·0125	0·0101	0·0085	0·0076	0·0028
	$\sigma_{\gamma WT}$	0·0018	0·0014	0·0012	0·0010	0·0009	0·0003
	$g_{\gamma WT}$	0·1514	0·1110	0·1603	0·1233	0·1480	0·4803
Ratio	$m_{\gamma WT}/m_\gamma$	—	0·58	0·58	0·58	0·58	0·61

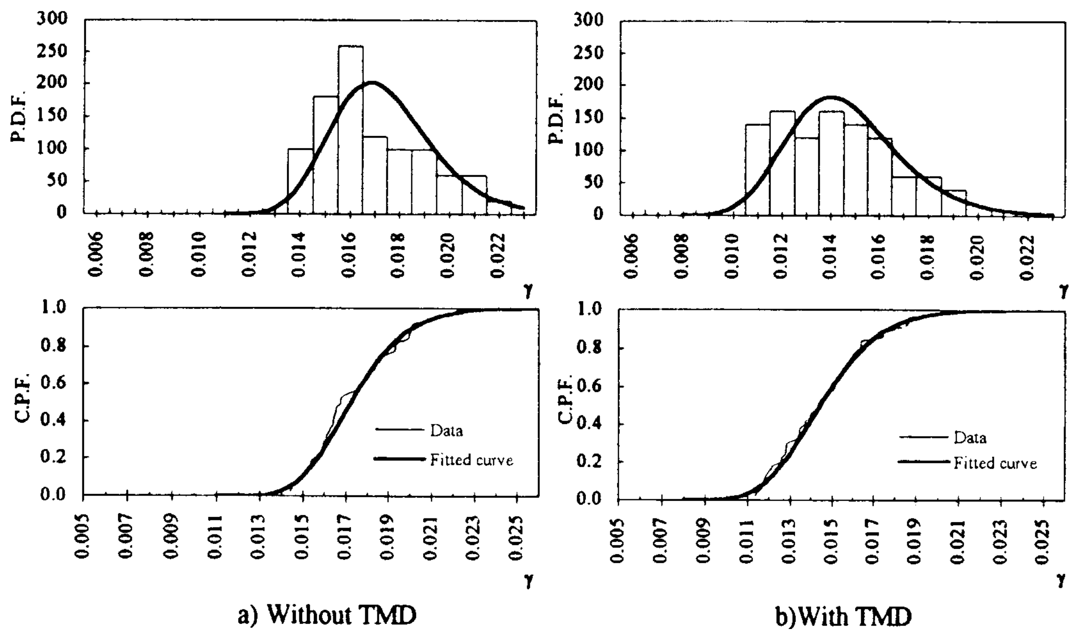

Figure 8. Statistics of the maximum storey drift (γ) of building designed with $Q = 2$, for an earthquake with return interval of 100 years

$Q = 4$, 2 and 1, respectively. This means that the reduction of the response is only 11 per cent for the non-linear system designed with $Q = 4$, and 42 per cent for the system that responds linearly ($Q = 1$).

5.2. Probability density function of the responses

Lognormal distribution functions were fitted to the data previously obtained for the cases $Q = 2$ and 4. The functions associated with $Q = 2$ are depicted in Figure 8. The left part of the

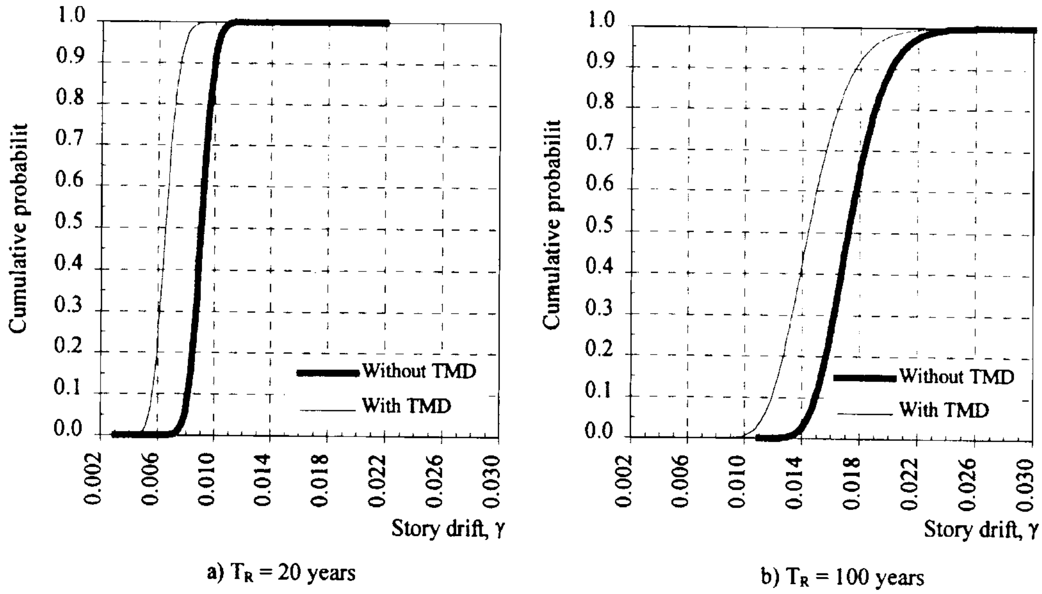

Figure 9. Cumulative Probability Functions (CPF) for return intervals of 20 and 100 years

figure corresponds to systems without TMD, and the right to those with TMD. The histograms of the observations and the fitted Probability Density Functions (PDF) are shown at the top; the Cumulative Probability Functions (CPF) are presented at the bottom. Two examples of CPF for $Q = 2$, associated with $T_R = 20$ and 100 years, are shown in Figures 9(a) and 9(b). In these figures, it can be seen that the probability of exceeding a prescribed storey drift δ is larger for structures without TMD than for those with TMD. Also, it can be seen that this reduction is more pronounced for structures subjected to motions with $T_R = 20$ years, than for those with $T_R =$ 100 years.

6. PROBABILITIES OF EXCEEDANCE OF THE LIMIT STATES

It is considered that a structural system fails when it can no longer satisfactorily perform the function for which it was designed. In general, the acceptance conditions related to the performance of each of those functions are expressed in terms of limit states imposed on adequate measures of the structural response. In this study, two values of the storey drifts were selected as limit states: 0·006 and 0·012. These coincide, respectively, with those recommended by the Mexico City Seismic Design Regulations (1993) for frames having walls rigidly connected to the structural frame (0·006), and for those where walls are isolated from the distorsions of the frame (0·012). However, if other values were used, the general conclusions of the study would not change significantly.

The probability of exceeding a limit state γ^* when an earthquake of a given intensity y acts is equal to $P(\gamma > \gamma^* | y) = P_F(y)$. Values of this probability for $\gamma^* = 0{\cdot}006$ and 0·012 were calculated for systems with a TMD and without it.

The values obtained for $Q = 2$ and 4 are shown in Figures 10(a) and 10(b), respectively. The figures at the left are for the limit state $\gamma > 0{\cdot}006$, and those at the right for $\gamma > 0{\cdot}012$. Continuous functions were fitted to the discrete values associated with the given return intervals. These functions, called here vulnerability functions, $P_F(y)$, are expressed in polynomial form. For example, for the case of systems designed with $Q = 2$, corresponding to the limit state $\gamma^* = 0{\cdot}012$, functions $P_F(y)$ have the following expressions:

(1) *System without TMD*:

$P_F(y) = 0{\cdot}0 \quad \text{for } y \leqslant 0{\cdot}79\ \text{m/s}^2$

$$P_F(y) = 730{\cdot}13\, y^5 - 3340{\cdot}0 y^4 + 5996{\cdot}59 y^3 - 5271{\cdot}60 y^2 + 2266{\cdot}65 y - 380{\cdot}93,$$

$$\text{for } 0{\cdot}79 \leqslant y \leqslant 1{\cdot}08\ \text{m/s}^2$$

$P_F(y) = 1{\cdot}0 \quad \text{for } y > 1{\cdot}08\ \text{m/s}^2$

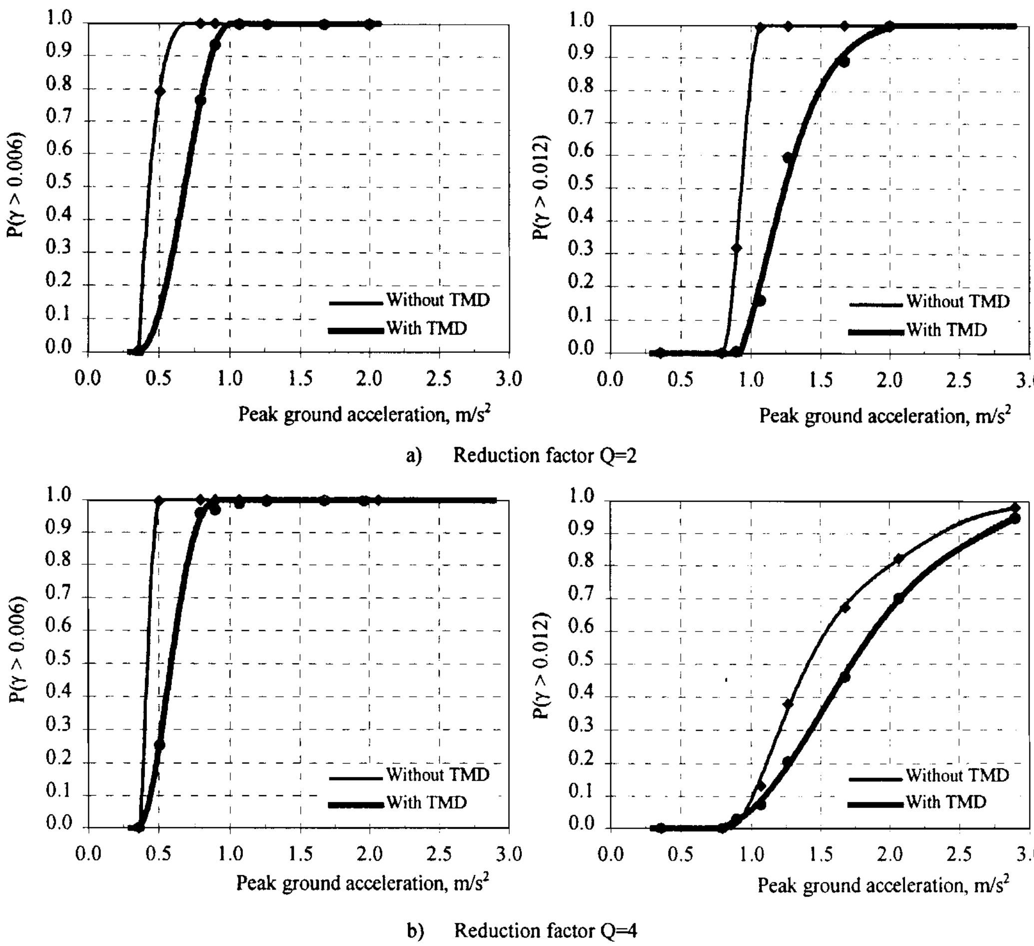

Figure 10. Probabilities of exceedance $P(\gamma > \gamma^*|y)$ for different reduction factors ($Q = 2$ and 4), and two limit states: $\gamma^* = 0{\cdot}006$ (left), and $\gamma^* = 0{\cdot}012$ (right)

(2) *System with TMD*:

$P_F(y) = 0{\cdot}0 \quad \text{for } y \leqslant 0{\cdot}92\ \text{m/s}^2$

$P_F(y) = 0{\cdot}6457y^6 - 7{\cdot}61y^5 + 36{\cdot}34y^4 - 89{\cdot}69y^3 + 119{\cdot}43y^2 - 79{\cdot}49y + 20{\cdot}47,$

$\text{for } 0{\cdot}92 \leqslant y \leqslant 2{\cdot}04\ \text{m/s}^2$

$P_F(y) = 1{\cdot}0 \quad \text{for } y > 2{\cdot}04\ \text{m/s}^2$

Again, the efficiency of the TMDs is shown to be higher for moderate than for high-intensity earthquakes.

7. FAILURE RATES OF THE SYSTEMS

The rate of failure per unit time of a structure with deterministically known properties is

$$v_F = \int_0^{y_M} -\frac{\partial v(y)}{\partial y} P(\gamma > \gamma^* | y)\, dy$$

where $v(y)$ is the rate of ocurrence of an intensity in excess of y, $P(\gamma > \gamma^* | y)$ is the probability that the structure reaches a limit state for a given y, and y_M is the expected maximum ground acceleration. In our case $v(y)$ is shown in Figure 2, the functions $P(\gamma > \gamma^* | y) = P_F(y)$ were obtained in the previous section, and $y_M = 2{\cdot}90\ \text{m/s}^2$ (see Figure 2). The final expression is the following:

$$v_F = -\int_0^{2\cdot 9} \left[\frac{0{\cdot}0913248[-1 + 0{\cdot}55088y^{0\cdot 56}]}{y^{2\cdot 26}} - \frac{0{\cdot}0223596}{y^{1\cdot 7}}\right] [P(\gamma \geqslant \gamma^* | y)]\, dy$$

This integration was performed numerically.

Table IV shows the values of the failure rates of the systems without TMD (v_F) and with TMD (v_{FWT}), associated with the limit states $\gamma^* = 0{\cdot}006$ and $0{\cdot}012$. It also shows the return intervals to failure $T_F = v_F^{-1}$ and $T_{FWT} = v_{FWT}^{-1}$, as well as their ratios T_{FWT}/T_F. Table IV indicates that the return interval to failure of the systems with TMD (T_{FWT}) is several times that of systems without it (T_F). According to these results, the efficiency of the TMD decreases as the Q and γ^* values increase.

Table IV. Ratios of return intervals to failure of systems without a TMD and with it

Reduction factor	$\gamma^* = 0{\cdot}006$					$\gamma^* = 0{\cdot}012$				
	Without TMD		With TMD		T_{FWT}/T_F	Without TMD		With TMD		T_{FWT}/T_F
	v_F	$T_F = 1/v_F$	v_{FWT}	T_{FWT}		v_F	$T_F = 1/v_F$	v_{FWT}	T_{FWT}	
$Q = 2$	0·1348	7·4	0·0719	13·9	1·88	0·0386	26·6	0·0212	47·1	1·78
$Q = 4$	0·1435	6·9	0·0894	11·2	1·61	0·0165	60·7	0·0119	83·7	1·37

8. CONCLUSIONS

8.1. Conclusions from the analysis of 22-storey frames

1. The parametric study performed on the 22-storey reinforced concrete frames shows that the maximum roof displacements are more strongly reduced when the frames are subjected to moderate motions, than to high-intensity ones. The former are associated with linear behaviour of the systems and the latter to non-linear behaviour. For the cases analysed, the maximum roof displacements of frames with TMD subjected to moderate motions can be about 68 per cent of those of the same frames without TMD; however, for the same frames subjected to high-intensity motions, the reductions are not higher than about 20 per cent (see Figures 4 (a) and (b)).
2. The increasing of damping of the TMD gives place to smaller roof displacements or to larger ones, depending on the frequency ratio and on the motion intensity. For moderate earthquakes and frequency ratios close to unity, increasing the damping of the TMD gives place to a smaller efficiency of the device; however, for frequency ratios equal or larger than 1.1, the effect of increasing the damping is beneficial to the reduction of the frame response. For systems subjected to high-intensity earthquakes the influence of damping is less well defined.
3. From the particular case of the frame with a TMD with specific values $R_m = 0{\cdot}03$ and $R_\omega = 1{\cdot}0$ (Figure 5) the following conclusions were reached:
   a. For systems under SCT-89 motion (linear behaviour of the frames):
   As the damping ratio ξ of the TMD increases, the maximum storey drifts developed at the bottom and at the middle of the frames also grow; however, at the top of the frame they decrease.
   b. For systems under SCT-85 motion (nonlinear behaviour of the frames):
   The influence of ξ on the interstorey drifts developed on frames with non-linear behaviour is not so clear as that observed in the linear systems.
   From these two paragraphs it is obvious that it is necessary to devote more efforts to study the influence of the damping of the TMD on systems subjected to ground motions.

8.2. Conclusions from the SDOF systems

1. The ratios of the mean maximum storey drifts on the systems with TMD and those without it (Tables I–III) increase as the return interval of the excitation is longer.
2. The ratios mentioned in the previous paragraph are larger for structures designed with a higher reduction factor Q than for those designed with a smaller factor.
3. The ratios of the return interval to failure of systems with TMD to those without TMD indicate that the effectiveness of TMDs is higher for systems with smaller nonlinearity, as well as for smaller limits states of the storey drifts γ^*.

8.3. Recommendations about the use of TMDs

The devices studied here can be efficiently used to reduce the interstorey drifts of buildings subjected to moderate earthquakes; however, their efficiency is highly reduced for systems developing nonlinear behavior, which generally occurs under high-intensity ground motions.

A possible solution could be to use more than one TMD on the roof of the structures in order to cover a wider range of periods of vibration.

ACKNOWLEDGEMENTS

Thanks are given to L. Esteva for the manuscript review. This study was done under grant 3683P-A of CONACyT.

REFERENCES

1. R. Villaverde and L. A. Koyama, 'Damped resonant appendages to increase inherent damping in buildings', *Earthquake Engng. Struct. Dyn.* **22**(6), 491–508 (1993).
2. Bernal, 'Influence of ground motion characteristic on the effectiveness of tuned mass dampers', *Proc. XI World Conf. on Earthq. Engng.*, Acapulco, Mexico, 1996.
3. S. E. Ruiz and L. Esteva, 'About the effectiveness of tuned mass dampers on nonlinear systems subjected to earthquakes', *Earthquake Resist. Engng. Struct., Adv. Earthquake Engng.* **2**, 311–320 (1997).
4. J. Alamilla, L. Esteva, J. García Pérez and O. Díaz, 'Evolutionary properties of stochastic models of earthquake accelerograms: their dependence on magnitude and distance', *J. Seismol.* (1999) submitted.
5. L. Esteva, O. Díaz, A. Terán and J. García-Pérez, 'Costos probables de daños causados por temblores en construcciones', *Report to AMIS* (Mexican Association of Insurance Institutions), Institute of Engineering, National University of Mexico, 1988.
6. M. Saiii and M. Sozen, 'Simple and complex modes for nonlinear seismic response of reinforced concrete structures', *Structural Research Series No. 405*, Department of Civil Engineering, University of Illinois at Urbana-Champaign IL, 1979.
7. S. Qi and J. Moehle, 'Displacement design approach for reinforced concrete structures subjected to earthquakes', *Report No. UBC/EERC-91-02*, University of California, Berkely, CA, 1991.
8. K. R. Collins, Y.-K. Wen and D. A. Foutch, 'Investigation of alternative seismic design procedures for standard buildings', *Structural Research Series No. 600*, Department of Civil Engineering, University of Illinois at Urbana-Champaign, IL, 1995.

